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Some Results on Restrained Triple Connected Domination in Line Graphs

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Abstract—*The concept of Restrained triple connected graph was introduced by G.Mahadevan in [3]. A set $D \subseteq V(L(G))$ is a Restrained Triple Connected Dominating (RTCD) set of $L(G)$, if every vertex not in D is adjacent to a vertex in D and to a vertex in V-D and $\langle D \rangle$ is triple connected. The RTCD number of a line graph $L(G)$ is denoted by $\gamma_{rtc}(L(G))$ is the minimum cardinality of a RTCD set of $L(G)$. In this paper, we discuss the graph theoretic properties of $\gamma_{rtc}(L(G))$ and its exact values for some standard graphs were obtained. The relationship with other parameter is also investigated. We characterize graphs which do not have γ_{rtc} set. We obtained the upper and lower bounds of $\gamma_{rtc}(L(G))$. We also proved the Nordhaus-Gaddum types of results of this parameter.*

Keywords: *Restrained triple connected domination number, Line graphs AMS Subject Classification: 05C*

1. INTRODUCTION

By a ***graph*** we mean a finite, simple, connected and undirected graph *G(V, E).* A subset *S* of *V* of a nontrivial graph *G* is called a *dominating set* of *G* if every vertex in $V - S$ is adjacent to at least one vertex in *S*. The *domination number* $\gamma(G)$ of *G* is the minimum cardinality taken over all dominating sets in *G*. A graph *G* is said to be triple connected if any three vertices lie on a path in *G*. A dominating set is said to be restrained dominating set if every vertex in V – S is adjacent to at least one vertex in *S* as well as another vertex in *V - S*. The minimum cardinality taken over all restrained dominating sets is called the restrained domination number and is denoted by $\gamma_r(G)$. The restrained dominating set is said to be restrained triple connected dominating set (RTCD), if the <S> is triple connected. The minimum cordiality taken over all the restrained triple connected dominating sets is called the **restrained triple connected domination number and is denoted by γ_{rtc} *(G).***

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With every non empty ordinary graph there is associated a graph L(G), called the line graph of G whose points are in one-to-one correspondence with the lines of G and such that two points are adjacent in L(G) if and only if the corresponding lines of G are adjacent.

1) Restrained Triple connected domination number in Line Graph

Definition 2.1: The restrained dominating set is said to be restrained triple connected dominating set (RTCD), if the <S> is triple connected. The minimum cordiality taken over all the restrained triple connected dominating sets is called the **restrained triple connected domination number and is denoted by γ_{rtc} *(G).*** With every non empty ordinary graph there is associated a graph L(G), called the line graph of G whose points are in one-to-one correspondence with the lines of G and such that two points are adjacent in L(G) if and only if the corresponding lines of G are adjacent.

Example:

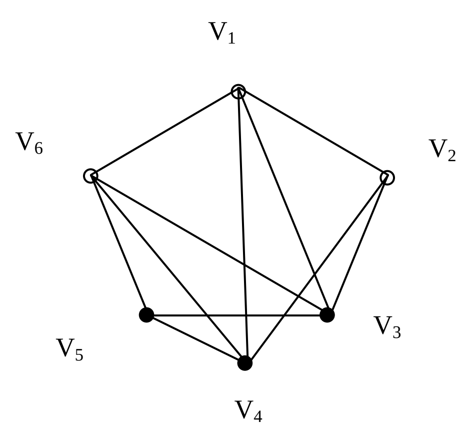


Fig. 2.1: $\gamma_{rtc}(L(K_4) = 3$

In this paper, we study the graph theoretic properties of $\gamma_{rtc}(L(G)$ and its exact values for some standard graphs were obtained. The relationship with other parameter is also investigated**3. Exact Values of $\gamma_{rtc}(L(G)$ for some standard graphs:**

1. $\gamma_{rtc}(L(C_p) = p - 2$
2. $\gamma_{rtc}(L((P_p) = q$
3. $\gamma_{rtc}(L((K_p) = 3$
4. $\gamma_{rtc}(L((k_{1,p}) = p, if\ p = 3,4$
5. $\gamma_{rtc}(L((C_{2,3}) = 3$
6. $\gamma_{rtc}(L(K_{1,p}) = 3, p \geq 5$
   ($L(K_{1,p}) = K_{p-1}$)
7. $\gamma_{rtc}(L(K_{1,p}) = p - 1$, p= 4,5
8. $\gamma_{rtc}(L((w_{1,p}) = 3$

Observations 3.1:

1. Every graph $L(G)$ without isolated vertex has a RTCD set as V($L(G) \geq 3$
2. The RTCD set contains all its pendent vertices, supports and the vertices of degree 2. Hence $\gamma_{rtc}(L(C_p) = p - 2$
3. For any connected graph with $\delta(G) \geq 2$ then $\gamma_{rtc}(L(G) \leq q$
4. For any connected graph G, $\gamma_{rtc}(G) = \gamma_{rtc}(L(G))$ and the equality satisfied for cycles and complete graphs
5. For any connected graph G, $\gamma_{rtc}(L(G) = q$ and the equality satisfied for a path graph
6. A RTCD number of a connected graph is isomorphic to its line graph if it is a cycle that is, $G \cong (L(G)$ if and only if G is a regular graph of degree 2.
7. If the line graph of a connected graph G is a tree then γ_{rtc} does not exist.
8. For any connected graph G with 3 vertices $\gamma_{rtc}(L(G)) = p$ if and only if $G \cong K_3$

Theorem 3.2: For any connected graph G with $p \geq 3$, then $3 \leq \gamma_{rtc}(L(G)) \leq q$.

Proof: The lower bound and the upper bound follow from the definition 2.1 and 2.2.

For $K_{1,p,}$ lower bound is attained and For Path graph, upper bound is attained.

Theorem 3.3: If every non end vertex of a tree T is adjacent to at least one end vertex then $\gamma_{rtc}(L(T)) \leq p + m$ where m is the number of end vertices of $L(T)$

Consider the tree of vertex greater than or equal to 4. Let X = $\{v_{1,}v_{2,..........,}v_m\}$ be the end vertices of T and if the non end vertex have exactly one end vertex then there should be an end vertex in $L(T)$ also, Every $k_{1,n}$ of T form a complete sub graph k_n of $L(T)$

Case(i) : $L(T)$ does not contain end vertex.

Let D= { $v_{1,}v_{2,..........,}u_{i,}$}, D $\geqq$ V($L(T)$) is the γ_{rtc} set of $L(T)$

Hence $\gamma_{rtc}(L(T))$ = D

Case(ii): $L(T)$ contains end vertex, X=$\{v_{1,} \dots \dots, v_m\}$

Hence $\gamma_{rtc}(L(T))$ = D ∪ $\{v_{1,}v_{2,..........,}v_m\}$ = D ∪ X

$|\gamma_{rtc}(L(T))|$ = $|D + m| \leq p + m$

Hence $\gamma_{rtc}(L(T)) \leq p + m$, where m be the end vertices of $L(T)$

Theorem 3.4: If every non end vertex of a tree T is adjacent to at least one end vertex then $\gamma_{rtc}(L(T)) = \lfloor p + 2 \rfloor$.

Proof: If every non end vertices of T is adjacent to at least one end vertex, then T is the combination of stars such as $K_{1,3}, K_{1,4} \dots \dots . . K_{1,n}$, thus L(T) is the combination of complete graph. Hence L(T) does not contain end vertices.

$$\gamma_{rtc}(L(T)) \leq q - 2$$

$$\gamma_{rtc}(L(T)) \leq p - 1 - 2$$

$$\gamma_{rtc}(L(T)) \leq p - 3$$

If some non end vertices of T is adjacent to at least one end vertex, then T is the combination of paths and stars. Obviously L(T) contains end vertices

$$\gamma_{rtc}(L(T)) \leq p + m$$

$$\gamma_{rtc}(L(T)) \leq p + 2$$

Hence
$\gamma_{rtc}(L(T)) = \lfloor p+2 \rfloor$.

Theorem 3.5: For any connected graph G, $\gamma_{rtc}(L(G) = \left\lceil \frac{p}{2} \right\rceil$ and the equality holds for caterpillar is which except the end vertices, the remaining vertices of degree ≥ 3.

Let D = $\{u_1, u_2, \ldots\ldots\ldots, u_m\} \subseteq$ V$(L(G)$ be the RTCD set of $(L(G)$. When V$(L(G)$ - D=0 then the result is obvious.

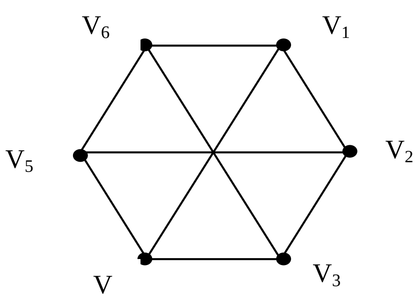
That is, V$(L(G)$ - D = p ≥ $p/2$

Suppose V$(L(G)$ - D ≥ 2 then V$(L(G)) - D$ Contains at least two vertices satisfies v-s is adjacent to S and V-S

There for $2n < p$ gives $n < p/2$

Thus $\gamma_{rtc}(L(G) = n \leq \left\lceil \frac{p}{2} \right\rceil$, Hence $\gamma_{rtc}(L(G) = \left\lceil \frac{p}{2} \right\rceil$

Theorem 3.7: For a connected Cubic graph G, $\gamma_{rtc}(L(G) = 3$ and the equality holds for G_1 and G_2. $\gamma_{rtc}(L(G) = \left\lceil \frac{q}{3} \right\rceil$ for $V(G) \geq 8$



G_1

Consider $V(G) = 6$

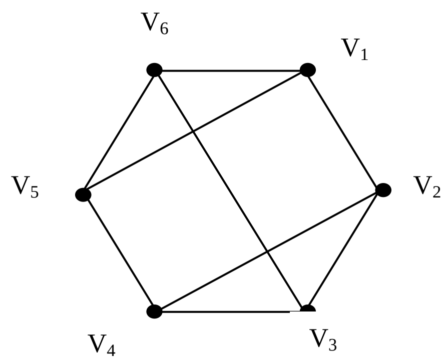
Let $V(G) = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ and

D = $\{v_1, v_2, v_3\}$

V-D = $\{v_4, v_5, v_6\}$

Case(i): $\langle S \rangle = P_3$, $\langle V - S \rangle = P_3$

V_1 is adjacent to V_5 and V_4 (or V_6) of V_1 is adjacent to V_5 and V_6 then V_3 is adjacent to V_4 and V_6 this V_2 is adjacent to V_4 then $G \cong G_1$, V_2 is adjacent to V_5 then $G \cong G_2$



G_2

Case(ii): $\langle S \rangle = K_3$, $\langle V - S \rangle = K_3$ if V_1 is adjacent to V_5 (or V_6 or V_4) then V_2 is adjacent to V_4 and V_3 is adjacent to V_6 . If V_1 is adjacent to V_6 then V_2 is adjacent to V_5 and V_3 is adjacent to V_4 the $G \cong G_1$

Consider $V(G) = 8$

Line Graph of Cubic graphs are the regular graph of order 4 and each vertex dominates 3 vertices of $(L(G)$ and $3m < q$ then $\gamma_{rtc}(L(G) = m \leq \left\lceil \frac{q}{3} \right\rceil$ by taking continuous dominating vertices then $\gamma_{rtc}(L(G) \geq \left\lceil \frac{q}{3} \right\rceil$

Observation: For any connected graph G, V$(L(G)) - D \leq \sum d(v_i)$

Theorem 3.8: All the end vertices of a connected graph L(G) is very γ_{rtc} set. Further more $\gamma_{rtc}(L(G) = \left\lceil \frac{q+1}{3} \right\rceil$.

Proof: Let $\{v_1, v_2, \ldots \ldots v_n\}$ be the vertex set of L (G) and $\{v_1, v_2, \ldots \ldots v_i\}$ are the end vertices of L (G). Let D be any RTCD set of L (G). If there exists$D_1 \leq V(L(G)) - D$ be also a RTCD set of L (G). Assume $v_1 \in V(L(G))$ and $v_1 \notin D_1$. Consider u, w be the vertices of V (L (G)) and $u, w \notin D_1$ and $v_1 \notin D_1$. Thus v_1 lies on every path of u-w path in L (G). Since the graph is triple connected then there is two possible cases. If $u, w \in D_1$then

$D = \{D_1 \cup \{u, w\} \cup v_1\}$ be the γ_{rtc} set of L(G).

$$= V(L(G)) - D + 3$$

$$2D = q + 3$$

$$D = \frac{q+3}{2}.$$

$|D| = |D_1| = 3$ gives D_1 is also be γ_{rtc} set of L(G).

If $u, w \notin D_1$ then $D = \{D_1 - \{u, w\} \cup v_1\}$

$= V(L(G)) - D + 3$

$$2D = q + 1$$

$$D = \frac{q+1}{2}$$

$|D| = |D_1| = 3$ Gives D_1 is also be γ_{rtc} set of L (G).

Hence $v_1 \in D_1$, thus the end vertex v_1 is in every γ_{rtc} set of L(G).

$$|D| = \frac{q+3}{2}, |D| = \frac{q+1}{2}$$

Hence $D = \gamma_{rtc}(L(G) = \left\lceil \frac{q+1}{3} \right\rceil$

Theorem 3.9: For any connected graph G, $\gamma_{rtc}(G) + \gamma_{rtc}(L(G) \leq p + 2$.

Proof: Let $D = \{v_1, v_2, \dots \dots v_n\}$ be the dominating set of G. If G contains end vertices, then $\gamma_{rtc}(G) = p$. If $\{e_1, e_2, \dots \dots e_k\}$ be the edge set of G, let $\{u_1, u_2, \dots \dots u_n\}$ be the vertex set of L(G). if L(G) has end vertices then $\gamma_{rtc}(L(G) = \{u_1, u_2, \dots \dots u_n\} = q$ be the RTCD set of L(G). if L(G) does not contain end vertices then $\gamma_{rtc}(L(G) = \{u_1, u_2, \dots \dots u_{n-2}\}$ be RTCD set of L(G). By line graphs, each vertex in L(G) is adjacent edges of G. hence every vertex of L(G) consistent to two adjacent vertices of G. thus $\gamma_{rtc}(G) + \gamma_{rtc}(L(G) \leq p + 2$.

Theorem 3.10: For any connected graph G, $\gamma((L(G)) + \gamma_{rtc}(L(G) \leq q$.

Proof: Let $\{u_1, u_2, \dots \dots u_n\}$ be the vertex set of L (G), deg $(v_i) \geq 2$.). Let D_1 be the RTCD set of L (G), if L (G) has end vertices then $D_2 = \gamma_{rtc}(L(G)) = \{u_1, u_2, \dots \dots u_n\} = q$ be the RTCD set of L(G). Since V (L(G)) = E (G) = q, $\gamma((L(G)) + \gamma_{rtc}(L(G) = D_1 \cup D_2 \leq q$.

Theorem 3.11: For any connected graph G,

1. $\gamma_{rtc}(L(G)) + \gamma_{rtc}(\overline{L(G)} \leq \lceil q + 1 \rceil$
2. $\gamma_{rtc}(L(G)) \cdot \gamma_{rtc}(\overline{L(G)} \leq \left\lceil \frac{q^2}{3} \right\rceil$ for any connected graph G
3. $\gamma_{rtc}(L(G)) \cdot \gamma_{rtc}(\overline{L(G)} \leq \left\lceil \frac{q^2}{2} \right\rceil$, if $G \cong P_n, G \ncong P_n, n = 2,3,4,5$

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